Nonlinear development of electron-beam-driven weak turbulence in an inhomogeneous plasma

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The self-consistent description of Langmuir wave and ion-sound wave turbulence in the presence of an electron beam is presented for inhomogeneous nonisothermal plasmas. Full numerical solutions of the complete set of kinetic equations for electrons, Langmuir waves, and ion-sound waves are obtained for an inhomogeneous unmagnetized plasma. The results show that the presence of inhomogeneity significantly changes the overall evolution of the system. The inhomogeneity is effective in shifting the wave numbers of the Langmuir waves, and can thus switch between different processes governing the weakly turbulent state. The results can be applied to a variety of plasma conditions, where we choose solar coronal parameters as an illustration, when performing the numerical analysis.

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I. INTRODUCTION

Electrostatic electron plasma waves are easily excited in controlled laboratory experiments [1,2], and in naturally occurring plasmas, by for instance low density electron beams [3–7]. The evolution of weakly nonlinear coherent wave phenomena has been extensively studied in simple geometries, and found to be well described by standard analytical models [8,9]. Cases where electron plasma waves, or Langmuir waves, are excited in plasmas made linearly unstable by electron beams [10] are particularly interesting as being one of the first kinetic instabilities where the nonlinear evolution described by quasilinear theory [11,12] was studied experimentally in a controlled laboratory experiment [13]. This beam excitation of plasma waves has proved particularly interesting in space plasmas, where high energy, low density electron bursts or beams are abundant. Full-wave numerical solutions for such problems have been carried out in unmagnetized as well as magnetized [14,15] plasmas, based on a standard and widely accepted theoretical model [16]. It is interesting that beam generated electron waves are observed by satellites in the interplanetary plasma near 1 AU [4], and in the ionosphere at high as well as lower altitudes by sounding rockets as well as satellites [4–6,17,18]. In these and similar cases, the waves appear as irregular or bursty in nature. A description aiming at a general full-wave analysis seems unrealistic in such cases, and a weak turbulence model has been suggested to give the evolution of the most important quantities, such as the averaged electron distribution function, and the wave spectrum [19–21]. The beam-plasma instability excites a spectrum of Langmuir waves, which in turn are damped or converted via nonlinear plasma processes, such as scattering off ions, $l+i \rightarrow l'+i'$ (nonlinear Landau damping off ions [19,22]), and decay into a Langmuir and ion-sound waves, $l \rightarrow l+s$, discussed previously [19,23]. Obviously, there are also other nonlinear processes that change the spectrum of Langmuir waves, but the processes mentioned are much more effective in affecting Langmuir waves in the applications of interest.

In the present work, we consider the case where energy density of the electron beam is much smaller than the thermal energy density of background plasma. The plasma waves and electron beam are described self-consistently by weak turbulence theory [19]. Quasilinear relaxation of an electron beam with the velocity $v_b \gg v_{Te}$ generates the primary Langmuir waves with wave numbers $\mathbf{k} \approx \omega_{pe}\mathbf{v}_b/v_b^2$ that causes the electron beam electron distribution function to relax toward a plateau in velocity space ranging from $v \sim v_b$ down to $v \sim v_{Te}$. Nonlinear processes, $l+i \rightarrow l'+i'$ and $l \rightarrow l'+s$, are effective in scattering of primary, beam generated waves with wave number $\mathbf{k}$, into secondary Langmuir waves with wave number $\mathbf{k}' \approx -\mathbf{k}$. The decay of a Langmuir wave also leads generation of ion-sound waves with $\mathbf{k}_s \approx 2\mathbf{k}$. The process repeats and produces the next generation of Langmuir waves. Every elementary cascade decreases the absolute value of Langmuir wave number by a small value $k_d^* = 2\sqrt{m_e/m_i}\sqrt{1+3T_i/T_e}/(3\lambda_{De})$ for decay and $k_s^* = 2\sqrt{m_e/m_i}\sqrt{T_i/T_e}/(3\lambda_{De})$ for scattering. Therefore, repeated scattering and decay processes lead to Langmuir waves being accumulated in the region of small values of $k \leqslant k^*$, the so-called Langmuir wave condensate. The decay processes are considered to be dominant process for nonisothermal plasma $T_e \gg T_i$, whereas scattering off ions is more important for isothermal plasma where ion-sound waves are heavily damped.

However, the presence of a plasma density gradient can change the spectrum of Langmuir waves significantly. In the presence of a plasma gradient, a given Langmuir wave with wave number $\mathbf{k}$ propagating in inhomogeneous plasmas changes its wave number to $\mathbf{k} \pm \Delta\mathbf{k}$, where $\Delta\mathbf{k}$ is determined by a plasma gradient. This effect may significantly change the Langmuir wave spectrum, and therefore slow down the nonlinear processes. If the plasma gradient is opposite to

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the direction of beam propagation, and the drift rate due to inhomogeneity is comparable with the rate of nonlinear processes, a steady state spectrum of Langmuir waves can appear. This case was considered in the case of isothermal plasma when the scattering off ions is the only nonlinear process in [24]. It was shown that the condensate of Langmuir waves appears, but in a shifted region of k and scattering off ions can be compensated by a plasma inhomogeneity. Moreover, a plasma inhomogeneity can change the quasilinear relaxation rate by shifting plasma waves from the unstable phase velocity region to the region where Langmuir waves are strongly damped by the beam electrons [25]. In a number of studies [26] it was found that a plasma inhomogeneity may suppress quasilinear relaxation, provided the beam density is sufficiently low. The positive plasma gradient can also lead to "self-acceleration" of beam electrons [26]. The importance of a plasma inhomogeneity demonstrated in previous studies [27] and the presence of inhomogeneities in most plasmas found in nature as well as in laboratories stimulates an investigation of the time evolution of beam-driven Langmuir turbulence in nonisothermal plasma. The principal aim of the present paper is to discuss the effect of density gradients on the wave dynamics and to demonstrate that even weak gradients can be of profound importance.

In the present study we perform a self-consistent investigation of the time evolution of weak turbulence in weakly inhomogeneous plasma. It is shown that a plasma inhomogeneity significantly modifies the scenario of weak turbulence. The disposition of the paper is as follows. In Sec. II we present a formulation of the problem, and present the basic equations. In Sec. III we present numerical results. In Sec. IV we summarize the main results of our investigations, while Sec. V contains our conclusions. In the present paper we attempt to consider physically realistic parameters, and have chosen those appropriate for electron beams and plasmas for the conditions in the solar corona. The problem is extremely time consuming numerically, and for this reason our investigations are restricted to one spatial dimension.

II. FORMULATION OF THE PROBLEM

We consider a beam of fast electrons and plasma waves within the limits of weak turbulence theory, when the energy density W of plasma waves with wave number k is much less than that of surrounding plasma,

$$\frac{W}{n\kappa T_e}<(k\lambda_{De})^2, \tag{1}$$

where n and T_e, are the electron plasma density and temperature, respectively, λ_{De} is the electron Debye length, while κ is Boltzmann's constant. In weak turbulence theory the evolution of electrons and waves is described by kinetic equations for an electron distribution function and spectral energy densities of plasma waves. The equations are essentially nonlinear, which significantly complicates the problem [19]. However, having in mind applications to low-β systems with relatively strong magnetic field when the energy density of magnetic field is much larger than the kinetic energy of fast electrons, but the magnetic field is not strong enough to magnetize the plasma waves, we can treat the system in one spatial dimension. The electron beams in the solar corona plasma are typical examples for such systems [28]. In these and similar cases, the plasma inhomogeneity along the beam propagation cannot be ignored.

We also assume that the variation dλ of the wave length λ of the Langmuir oscillations is a small, i.e.,

$$\left|\frac{\mathrm{d}\lambda}{\mathrm{d}x}\right|\ll 1, \tag{2}$$

or in other words, we describe wave propagation in geometrical optics (WKB) approximation [21,29]. Using the fact that the frequency of a Langmuir wave does not change during its propagation in the plasma, we readily derive the condition for applicability of the WKB approximation from Eq. (2),

$$\frac{v}{|L|}\ll 3\omega_{pe}(x)\left(\frac{v_{Te}}{v}\right)^2, \tag{3}$$

where

$$L\equiv\omega_{pe}(x)\left(\frac{\partial\omega_{pe}(x)}{\partial x}\right)^{-1} \tag{4}$$

is the scale of the local inhomogeneity, ω_{pe} is the local plasma frequency, and v_{Te}, $v=\omega_{pe}/k$ are the electron thermal and wave phase velocities, respectively.

The time evolution of the average velocity distribution $f(v,t)$ is described by quasilinear theory, which basically describes a diffusion process in velocity space, where the diffusion coefficient is self-consistently determined by the spectrum of the Langmuir waves [11,12]. The velocity distribution is assumed to be the same all over the relevant part of space, and in those cases where we deal with an inhomogeneous plasma, the density variation is contained in a coefficient. The growth and damping of plasma waves is accounted for by the standard Landau prescription and derived from the derivative of the velocity distribution at the phase velocity of the waves. The evolution of the wave spectra is described by the wave kinetic equation basically having the form

$$\frac{\partial W_k}{\partial t}+v_g\frac{\partial W_k}{\partial x}-\frac{\partial\omega_k}{\partial x}\frac{\partial W_k}{\partial k}=St, \tag{5}$$

which for $St=0$ is the Liouville equation. Equation (5) can be postulated from basic physical arguments [20,21], or be derived [30] from more basic equations [16]. In the dynamic equation (5), we can heuristically interpret W_k as a space time varying distribution of wave packets, each with a carrier wave number k, propagating with a corresponding group velocity $v_g\equiv\partial\omega_k/\partial k$, and subject to an effective force $-\partial\omega_k/\partial x$. In the present case we have $\partial\omega_k/\partial x\approx\partial\omega_{pe}/\partial x$, and this force is directly related to the gradient in plasma density through the density dependence of ω_{pe}. The wave packets constituting W_k are accelerated in the direction towards smaller densities while the carrier wave number increases. The term St on the right-hand side of Eq. (5) accounts for sources and sinks, together with effects that redistribute energy within the spectrum. With the foregoing assumptions (2), we can assume $\partial\omega_{pe}/\partial x \approx \omega_{pe}/L$, where ω_{pe} is now a constant plasma frequency obtained at a reference position in the center of the system. Apart from a negligible correction of the relative order $k\lambda_{De}$, this effective force is independent of wave number, and it will not induce any spatial variations in an initially uniform distribution W_k. For that case we consequently have $\partial W_k/\partial x = 0$. With these assumptions we simplify Eq. (5) in the following. As far as St is concerned, we have the linear instability acting as a source, and Landau damping as a sink of wave energy. Decay and nonlinear Landau damping act as sinks of wave energy, but equally important, these effects redistribute the energy within the spectrum W_k. The mathematical expressions for these latter effects are standard [19,23].

The basic equations are treated as an initial value problem (just as in related recent studies [31–33]). This gives a simplified alternative to the full problem, retaining at the same time the physics being important here. Using the assumptions mentioned above we can write the system of kinetic equations of weak turbulence theory

$$\frac{\partial f}{\partial t} = \frac{4\pi^2 e^2}{m^2}\frac{\partial}{\partial v}\frac{W_k}{v}\frac{\partial f}{\partial v}, \tag{6}$$

$$\frac{\partial W_k}{\partial t} - \frac{\omega_{pe}}{L}\frac{\partial W_k}{\partial k} = \frac{\pi\omega_{pe}^3}{nk^2}W_k\frac{\partial f}{\partial v} + St_{ion}(W_k) + St_{decay}(W_k, W_k^s), \tag{7}$$

$$\frac{\partial W_k^s}{\partial t} = -2\gamma_k^s W_k^s - \alpha\omega_k^{s2}\int\left[\frac{W_{k'-k}}{\omega_{k'-k}}\frac{W_k^s}{\omega_k^s} - \frac{W_{k'}}{\omega_{k'}}\left(\frac{W_{k'-k}}{\omega_{k'-k}} + \frac{W_k^s}{\omega_k^s}\right)\right]\delta(\omega_{k'} - \omega_{k'-k} - \omega_k^s)dk', \tag{8}$$

$$\begin{aligned} St_{decay}(W_k, W_k^s) = \alpha\omega_k\int \omega_{k'}^s\Bigg\{&\left[\frac{W_{k-k'}}{\omega_{k-k'}}\frac{W_{k'}^s}{\omega_{k'}^s} - \frac{W_k}{\omega_k}\left(\frac{W_{k-k'}}{\omega_{k-k'}} + \frac{W_{k'}^s}{\omega_{k'}^s}\right)\right]\delta(\omega_k - \omega_{k-k'} - \omega_{k'}^s) \\ &- \left[\frac{W_{k+k'}}{\omega_{k+k'}}\frac{W_{k'}^s}{\omega_{k'}^s} - \frac{W_k}{\omega_k}\left(\frac{W_{k+k'}}{\omega_{k+k'}} - \frac{W_{k'}^s}{\omega_{k'}^s}\right)\right]\delta(\omega_k - \omega_{k+k'} + \omega_{k'}^s)\Bigg\}dk', \end{aligned} \tag{9}$$

$$St_{ion}(W_k) = \beta\omega_k\int\frac{(\omega_{k'} - \omega_k)}{v_{Ti}|k-k'|}\frac{W_{k'}}{\omega_{k'}}\frac{W_k}{\omega_k}\exp\left[-\frac{(\omega_{k'} - \omega_k)^2}{2v_{Ti}^2|k-k'|^2}\right]dk', \tag{10}$$

where

$$\alpha = \frac{\pi\omega_{pe}^2(1+3T_i/T_e)}{4n\kappa T_e}, \quad \beta = \frac{\sqrt{2\pi}\,\omega_{pe}^2}{4n\kappa T_i(1+T_e/T_i)^2}, \tag{11}$$

$$\gamma_k^s = \sqrt{\frac{\pi}{8}}\omega_k^s\left\{\frac{v_s}{v_{Te}} + \left(\frac{\omega_k^s}{kv_{Ti}}\right)^3\exp\left[-\left(\frac{\omega_k^s}{kv_{Ti}}\right)^2\right]\right\}, \tag{12}$$

where γ_k^s is the damping rate of ion-sound waves, $v_s = \sqrt{\kappa T_e(1+3T_i/T_e)/m_i}$ is sound speed, $f(v,t)$ is the averaged electron distribution function, $W(k,t)$ and $W^s(k,t)$ are the spectral energy densities of Langmuir waves and ion-sound waves, respectively. $W(k,t)$ plays the same role for waves as the electron distribution function for particles. The system (6) and (7) describes the resonant interaction $\omega_{pe} = kv$ of electrons and Langmuir waves. The last term on the left-hand side of Eq. (7) represent the shift in wave number induced by the plasma inhomogeneity.

In the right-hand side of Eqs. (6) and (7) we omitted terms accounting for spontaneous emissions, because they are small in comparison to the ones retained. It should be also noted that small collisional terms are not included either. The system reaches steady state before collisional effects become important.

III. NUMERICAL RESULTS

We consider an initial value problem when all initial energy is accumulated in electron beam. The electron distribution function of electrons at the initial time moment $t=0$ is the combination of fast electrons and background Maxwellian electrons

$$f(v,t=0)=\frac{n_b}{\sqrt{\pi}\Delta v_b}\exp\left(-\frac{(v-v_b)^2}{\Delta v_b^2}\right)+\frac{n}{\sqrt{2\pi}v_{Te}}\exp\left(-\frac{mv^2}{2\kappa T_e}\right), \qquad (13)$$

where n_b, Δv_b are the beam density and the electron beam thermal velocity. The initial spectral energy density is thermal,

$$W(k,t=0)\approx\frac{\kappa T_e}{2\pi^2\lambda_{De}^2}, \qquad (14)$$

where T_e is the electron temperature of the surrounding plasma, and λ_{De} is the electron Debye length. The system of kinetic equations is reduced to dimensionless form and integrated using finite difference schemes. For the numerical time integration we used a method similar to the one used in Ref. [34] and the quasilinear terms are integrated using methods described in [35]. Previous results [34,35] are found in full agreement with the corresponding limiting cases of our calculations.

The electron beam and plasma parameters are taken typical for the conditions in the solar corona [36,37]. We use $v_b=12.8\,v_{Te}=5\times10^9$ cm/s, $\Delta v_b=0.3v_b$, $T_i/T_e=0.3$, $T_e=10^6$ K, $n_b=50$ cm^{-3}, and $\omega_{pe}/2\pi=200$ MHz. The plasma inhomogeneity is given by a constant plasma gradient, which can have either sign. Six different cases have been considered: the strongest plasma inhomogeneity considered is $L=\pm1\times10^8$ cm, a medium plasma inhomogeneity $L=\pm5\times10^8$ cm, and a weak plasma inhomogeneity $L=\pm1\times10^9$ cm. Such plasma inhomogeneity might exist in the low corona [38], in ionosphere [39], and in the solar corona due to small scale inhomogeneity [40].

A. Homogeneous plasma

As a reference case we first considered the evolution of Langmuir turbulence in a homogeneous plasma, corresponding to $L\rightarrow\infty$. Results are shown in Fig. 1, demonstrating that the fastest process in the system is quasilinear relaxation, which drives energy out of a beam and into Langmuir waves. As a result of quasilinear relaxation the electron distribution function rapidly flattens, building a plateau from $15v_{Te}>v>4v_{Te}$ for $t\approx0.02$ s, which is close to a quasilinear time $\tau\approx n/\omega_{pe}n_b$. The beam driven Langmuir turbulence has a bright maximum at $k\lambda_{De}=v_{Te}/v_b\approx0.087$. The primary Langmuir waves generated by a beam are subject to decay into a secondary Langmuir waves and ion-sound waves. The decay of Langmuir waves starts from the maximum of $W(k)$ and produce back-scattered Langmuir waves with maximum at $k\lambda_{De}=-0.065$. Secondary Langmuir waves reaches its maximum at $t=0.04$ s. In their turn secondary Langmuir waves produce a new generation of scattered Langmuir waves with maximum at $k\lambda_{De}\approx0.044$. It is obvious that an absolute value of k decreases with each act of decay by $k_d^*\lambda_{De}\approx0.021$. Thus, step by step the maximum of the Langmuir wave distribution approaches the region of wave numbers where decay is prohibited, i.e., the region $k<k_d^*/2$. However, since the spectrum of Langmuir waves generated by a beam is broad in wave numbers, the decay continues in wave number regions where the level of Langmuir waves is relatively low. The rate of three wave decay is proportional to the intensity of Langmuir waves and, therefore, at a given moment we see a few generations of Langmuir waves simultaneously (Fig. 1). Thus, each generation of Langmuir waves appear as a parabolic structure in that part of Fig. 1, which shows the wave number distribution as a function of time.

The ion-sound turbulence has a low intensity due to the strong ion Landau damping in a plasma with $T_i/T_e=0.3$. However, ion-sound waves are generated during each decay. The decay of a primary waves is marked by ion-sound waves with maximum at $k\lambda_{De}\approx0.15$, which corresponds to $k_s=2k-k_d^*$. The maximum of ion-sound waves due to the decay of secondary waves is seen at $k_s\lambda_{De}\approx0.125$.

The energy of fast electrons, i.e., the beam energy, is given by

$$E_e(t)=\int_{4v_{Te}}^{\infty}mv^2f(v,t)dv/2, \qquad (15)$$

the total energy of waves

$$E_{l,s}(t)=\int_{-\infty}^{\infty}W^{l,s}(k,t)dk, \qquad (16)$$

the energy of waves propagating along and against the beam

$$E_{l,s}^{+,-}(t)=\pm\int_0^{\pm\infty}W^{l,s}(k,t)dk. \qquad (17)$$

The corresponding distributions are shown in Fig. 1. The lower integration limit in Eq. (15) of course chosen somewhat arbitrarily, and the energy and density of a beam obtains a somewhat arbitrary value. At later times, E_e may therefore exceed its initial value, when some of the background electrons are accelerated to velocities above the value $4v_{Te}$ chosen in Eq. (15). In the presentation here, the distribution function is truncated at a level of 3 in the normalized units, and, therefore, the background plasma appears as a black band. To interpret the gray scale in, for instance, the Langmuir wave number distribution as a function of time, the energy distribution $E_l(t)/E_0$ can be used as guide.

The energy distribution in the system follows the well-known scenario [23]. In the present case, one-fifth of the initial beam energy is transformed into wave energy E_e^+ of Langmuir waves along the beam propagation during the quasilinear relaxation. However, the decay redistributes the energy between the primary waves and scattered waves. As a result the typical oscillations of wave energy $E_l^{\pm}$ along and opposite to the beam direction are clearly seen in Fig. 1(b). The total energy of Langmuir waves is almost constant, displaying the conservation of energy in the three-wave decay. Due to the strong damping of ion-sound waves, the energy of density oscillations is a small fraction of the initial beam energy. Ion-sound waves generated in the decay are rapidly

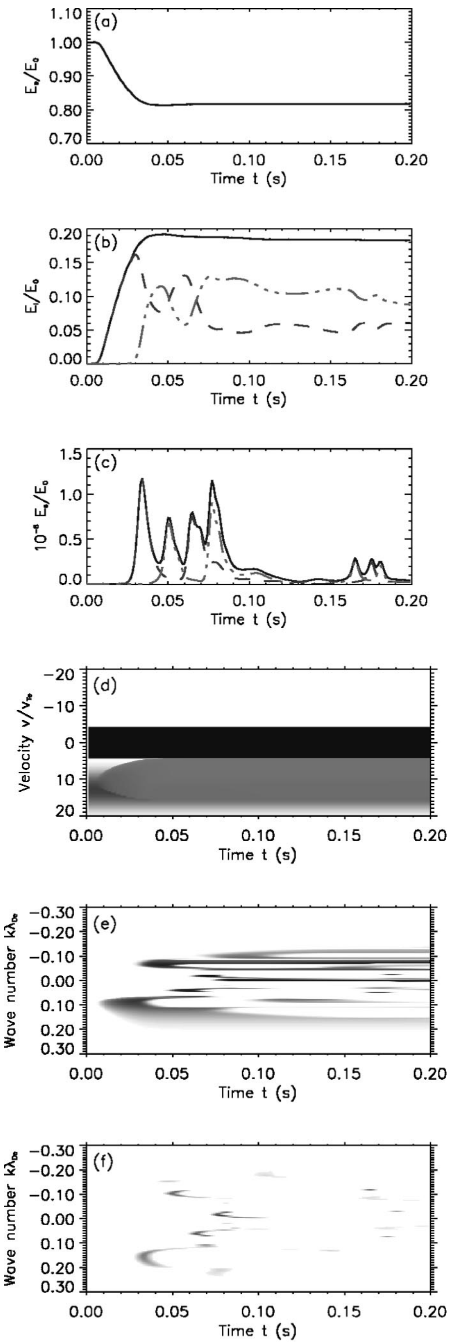

FIG. 1. Homogeneous density plasma. The normalized energy of electrons $E_e(t)/E_0$, is shown in (a), the normalized energy in the Langmuir waves $E_l(t)/E_0$ in (b), and correspondingly for the ion-sound waves $E_s(t)/E_0$ are shown in (c). The energy of waves along $E^+_{l,s}/E_0$ and against $E^+_{l,s}/E_0$ the beam propagation is given by dashed and dash-dotted lines, respectively. The time evolution of the spectral distribution of electrons $f(v,t)v_b\sqrt{\pi}/n_b$ is shown in (d); for Langmuir waves $W(k,t)\omega_{pe}/mn_bv_b^3$ in (e); and ion-sound waves $W(k,t)\omega_{pe}/mn_bv_b^3$ is shown in (f). The disposition of the following figures is the same as here.

absorbed by linear ion Landau damping, which is significant for the temperature rations in the present problem. Therefore, the amplitude of ion-sound wave oscillations is small, see Fig. 1(f). In order to interpret the gray-scale intensity levels, the energy curves in Figs. 1(b) and 1(c) can be used for an estimate. Obviously, the system tends towards a steady state solution. The final energy distribution of Langmuir waves is not symmetric in k. The energy of waves propagating against beam direction, E_l^-, is one-half of that propagating along the beam, E_l^+. This is simply due to back absorption of Langmuir waves by the beam.

The scattering off ions seems to be negligible during the initial redistribution of Langmuir waves. This trivial result proves the well-known fact that decay process is the fastest process changing the Langmuir spectrum in nonisothermal plasma. However, the scattering off ions plays a critical role at later stages, when the Langmuir waves are accumulated in the region of $|k|\leqslant k_d^*/2$. The reason for this is that decay effectively generates high level of Langmuir waves in the region of small k where scattering off ions more effective. Moreover, since decay is impossible for $k_s^*/2\leqslant|k|\leqslant k_d^*/2$, scattering off ions seems to be the only process that continues to contribute to the Langmuir wave condensate. To prove the role of scattering off ions, we turned off the corresponding term in our calculations. These results are presented in Fig. 2. Some differences are found between Figs. 1 and 2, e.g., details of the spectra of Langmuir waves presented for these two cases differ for $t>0.05$ s.

B. Strong plasma inhomogeneity

Since overall evolution of the system strongly depends on the plasma gradient value we consider three different length scales for the inhomogeneity separately.

The main results for strong plasma gradient are presented in Figs. 3 and 4. In this case the shift of the spectrum, due to plasma inhomogeneity, is stronger than any nonlinear process in the system, and the plasma inhomogeneity suppresses all nonlinear processes, but it is not strong enough to arrest the quasilinear relaxation. Independent on the sign of the plasma gradient L, the evolution of the Langmuir turbulence is very limited in time, while the physical processes are different for positive and negative gradients.

In the case of a positive plasma density gradient, see Fig. 4, all plasma waves generated during the relaxation are absorbed back by the beam at time $t=0.02$ s. We see that the drift of Langmuir waves in k space is so fast that nonlinear processes are not observable. No signs of ion-sounds waves are observed neither. For the time comparable with the quasilinear time, Langmuir waves are shifted from the generation region to absorption region. As a result, accelerated electrons are seen in Fig. 4. The plateau is now formed from $20\,v_{Te}>v>4\,v_{Te}$. The amount of energy released from a beam is a small fraction of the beam energy. Therefore, this case is specially interesting in terms of stabilization of quasilinear relaxation by a plasma inhomogeneity. This limiting case has been considered in the literature [25,26,41] and a simplified solution can be found in this case. Thus, the strong positive gradient $L=1\times10^8$ cm leads to self-acceleration of electrons in a beam, consistent with observations reported in [4], for instance.

In the case of a negative plasma gradient (decreasing plasma density), Langmuir waves are also effectively ab-

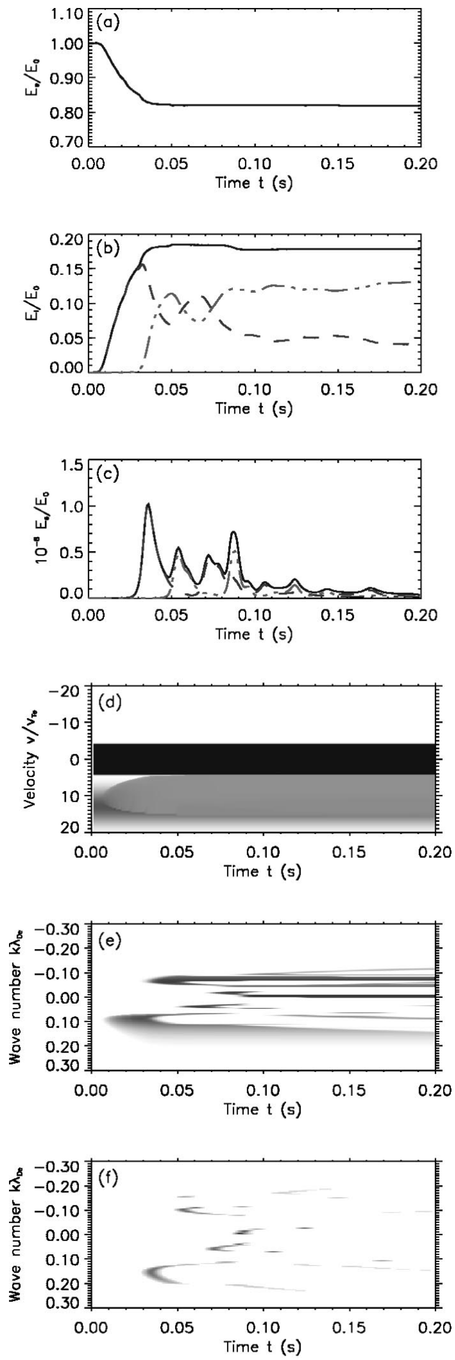

FIG. 2. The same as Fig. 1, but the scattering off ions is switched off.

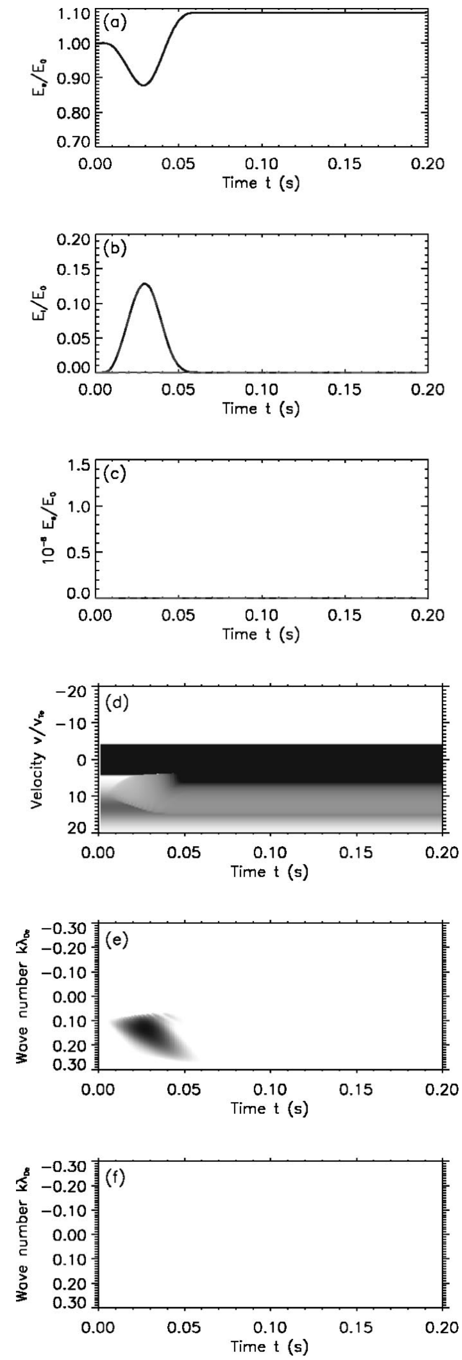

FIG. 3. The same as Fig. 1, but for decreasing density plasma along beam propagation. Strong plasma inhomogeneity $L=-1\times10^{8}$ cm.

sorbed back by electrons. Contrary to the case with $L>0$, Langmuir waves are now absorbed by the background electrons with velocities close to thermal velocities (classical Landau damping). As a result of this Landau damping, we see the appearance of accelerated electrons in the range $4\,v_{Te}<v<9\,v_{Te}$. The electron distribution function of these electrons is a decreasing function of velocity. Thus, due to the inhomogeneity, plasma waves play a role of Dreicer field extracting electrons from the background Maxwellian distribution. Figure 4 demonstrates that the amount of energy of electrons with $v>4\,v_{Te}$ is greater than it was before the quasilinear relaxation. The spectral energy density of Langmuir waves reaches its maximum value at $k\lambda_{De}\approx0.15$, which is quite different from the beam resonant region of k, where generation of Langmuir waves takes place, see Fig. 5. It is also worth noting that the amount of energy pumped into Langmuir waves is much larger than in the case of a positive plasma gradient and comparable with the case of a homogeneous plasma. The amount of energy obtained by Langmuir waves is about 13% of initial beam energy. The maximum is reached at time $t=0.03$ s. Similar to the case of a positive

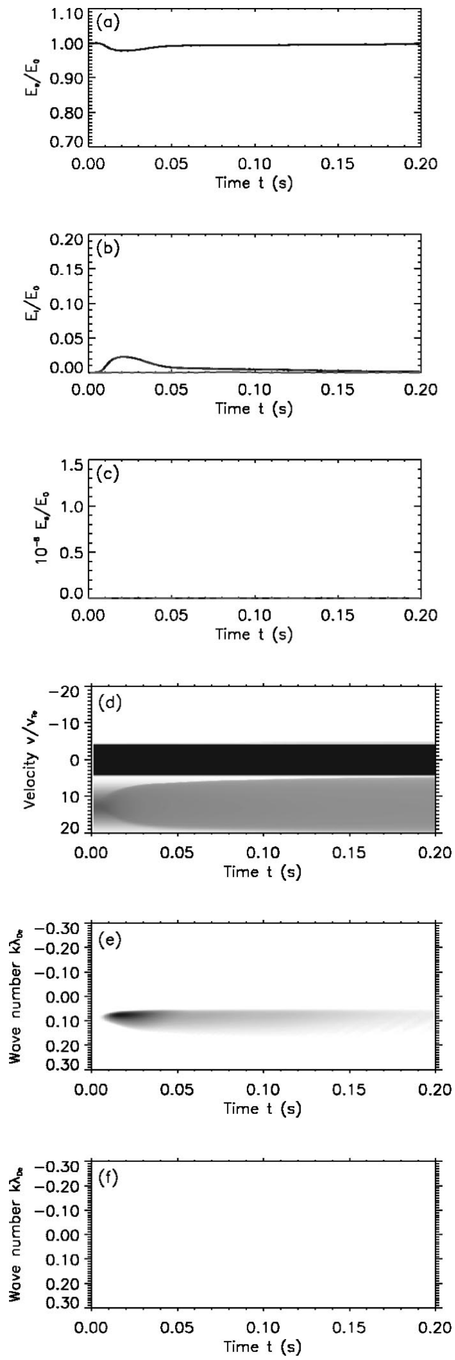

FIG. 4. The same as Fig. 1, but for an increasing plasma density. Strong plasma inhomogeneity, with $L = 1\times10^8$ cm.

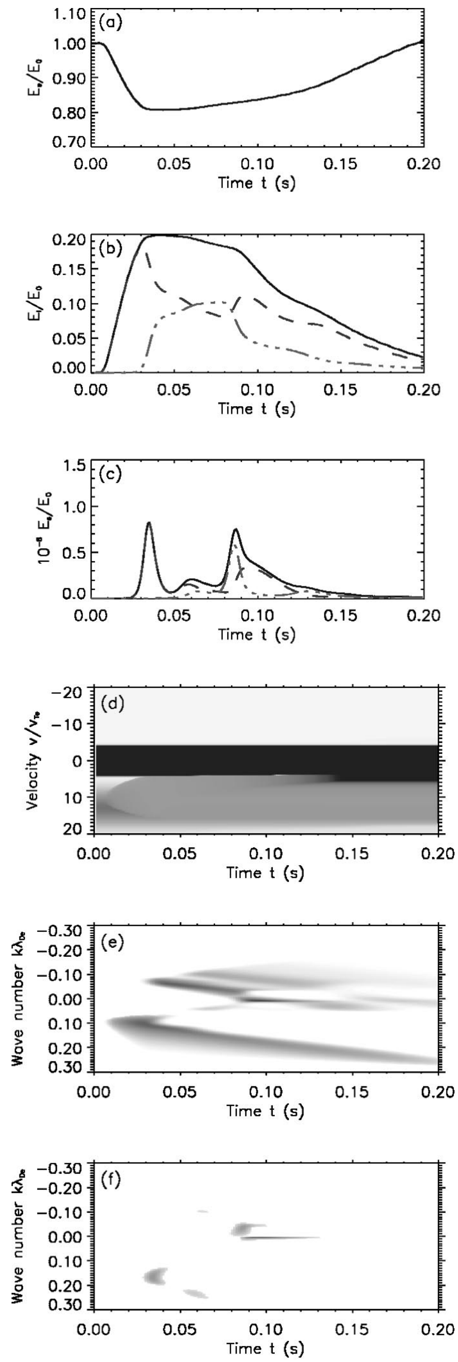

FIG. 5. The same as Fig. 1 but for an decreasing plasma density. Medium plasma inhomogeneity $L = -5\times10^8$ cm.

plasma gradient, there is no sign of either back-scattered Langmuir nor ion-sound waves. Thus, the plasma inhomogeneity suppresses any further nonlinear development of weak turbulence.

C. Medium plasma inhomogeneity

Decreasing the value of the plasma gradient in our system we can stimulate a further development of weak turbulence. For $|L| = 5\times10^8$ cm a single decay is observable in Figs. 5 and 6. The primary waves, as in the case of homogeneous plasma, have their maximum close to $k\lambda_{De} = 0.086$. The secondary waves are accumulated near $k\lambda_{De} = -0.065$. The shift of the Langmuir wave spectrum due to plasma inhomogeneity is comparable with three-wave decay of secondary waves. Therefore, influenced by plasma inhomogeneity, the secondary waves move toward smaller k for $L > 0$ and toward larger k for $L < 0$. The sign of a plasma gradient determines the spectrum of Langmuir waves and the operating physical processes. However, independently on the sign of the plasma gradient (Figs. 5 and 6) the system approaches a steady state faster than in case of a homogeneous plasma.

We consider the case of decreasing plasma density. One of the conspicuous features of the turbulence evolution in inho-

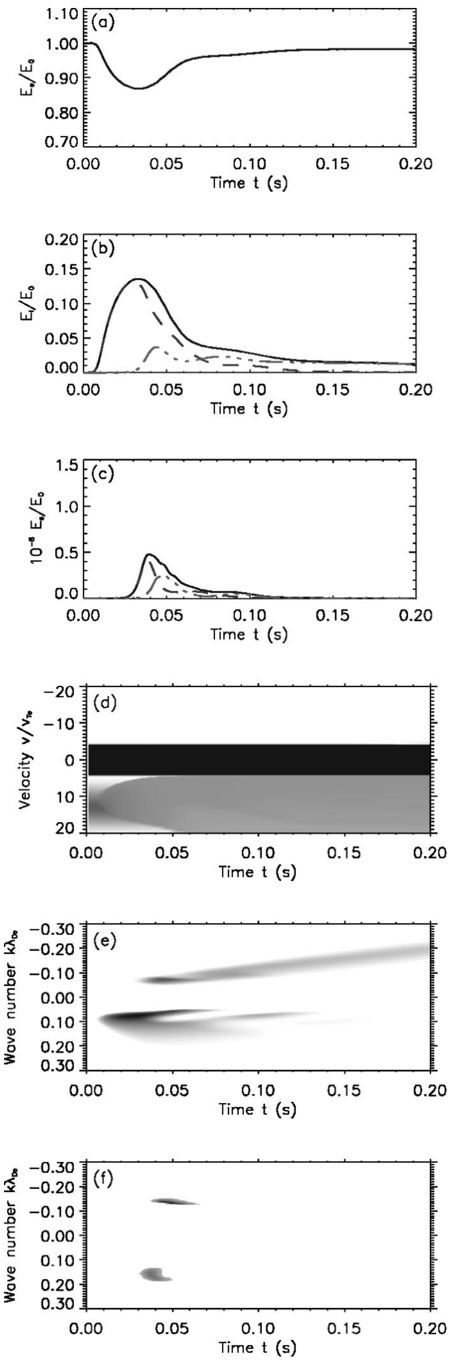

FIG. 6. The same as Fig. 1 but for an increasing plasma density. Medium plasma inhomogeneity $L=5\times10^{8}$ cm.

mogeneous plasma is the existence of quasisteady states. The spectrum of Langmuir waves remains almost constant at the time scale greater than the time required for a decay. To obtain steady state, one needs a constant source of Langmuir waves [24]. In Fig. 5 we see that during some time the values of $E_l^{\pm}$ do not follow the typical oscillations associated with a decay, as seen for instance in Fig. 1. In homogeneous plasma these values undertake oscillations with a period defined by the intensity of waves. The formation of the quasisteady state is explained by the following observations. Due to three-wave interaction, Langmuir waves spread over k space, with each scattering resulting in lower intensities for a given k. The rate of decay is essentially determined by the intensity of the Langmuir waves and generally decreases with each decay. At some instant, the rate of decay becomes comparable with the effect induced by the plasma inhomogeneity. From this time, $t\approx0.05$ s, the decay is suppressed by the shift due to the plasma density gradient. With time, this equilibrium is broken at $t\approx0.09$ s, and the oscillations proceed. This quasisteady state is also clearly seen in the spectrum of ion-sound waves. The ion-sound waves corresponding to the decay of backscattered Langmuir waves appear only after breaking of the equilibrium. It should be noted that the stabilization of the decay instability by the plasma inhomogeneity has not been considered before. The stabilization of a decay is somewhat analogous to stabilization of scattering off ions, the only effective process for an isothermal plasma. The stabilization of scattering off ions by a plasma inhomogeneity has been considered previously [24]. In a more general cases, when both nonlinear processes are allowed, the compensation of a decay by a plasma inhomogeneity enables scattering off ions to be seen at early stages of turbulence. The quasisteady state spectrum of Langmuir turbulence slowly changes due to scattering off ions.

The opposite case, with positive sign of plasma gradient, $L>0$, leads to Landau damping of back-scattered waves on thermal electrons with negative velocity. Indeed, as a result of absorption, accelerated electrons are seen in the range $-8\,v_{Te}<v<-4\,v_{Te}$ (Fig. 6). Further development of wave turbulence is suppressed and a quasisteady state is not formed. Similar to the large plasma gradient case, with $L>0$, the significant part of primary Langmuir waves are reabsorbed by the beam. The maximum velocity of the plateau, see Fig. 6, is clearly larger than in the case of a homogeneous plasma.

D. Weak plasma inhomogeneity

When the plasma inhomogeneity is decreased, the overall pictures tends to be more and more similar to the case of homogeneous plasma. However, there are some features, which makes the spectrum different from homogeneous plasma (Figs. 7 and 8). The weak plasma inhomogeneity effectively governs the turbulence in the range of small k.

The negative plasma gradient (Fig. 7) again leads to some quasisteady states, but for $L=-1\times10^{9}$ cm it requires a few acts of decay to reach compensation of three-wave decay by plasma inhomogeneity. The noticeable part of primary Langmuir wave energy goes directly to background plasma. The other waves experience decay and are then absorbed by the beam. We also see an appearance of accelerated electrons as in the case of increasing plasma density (Fig. 7). However, the physics behind this process is different. Both primary Langmuir waves and back-scattered secondary waves are now absorbed by the beam. Thus, a relatively weak plasma inhomogeneity, with $L=1\times10^{9}$ cm prevents further decay of Langmuir waves.

The notable point is the influence of a plasma inhomogeneity in the region of small k, where decay is prohibited but scattering off ions continues building of Langmuir wave con-

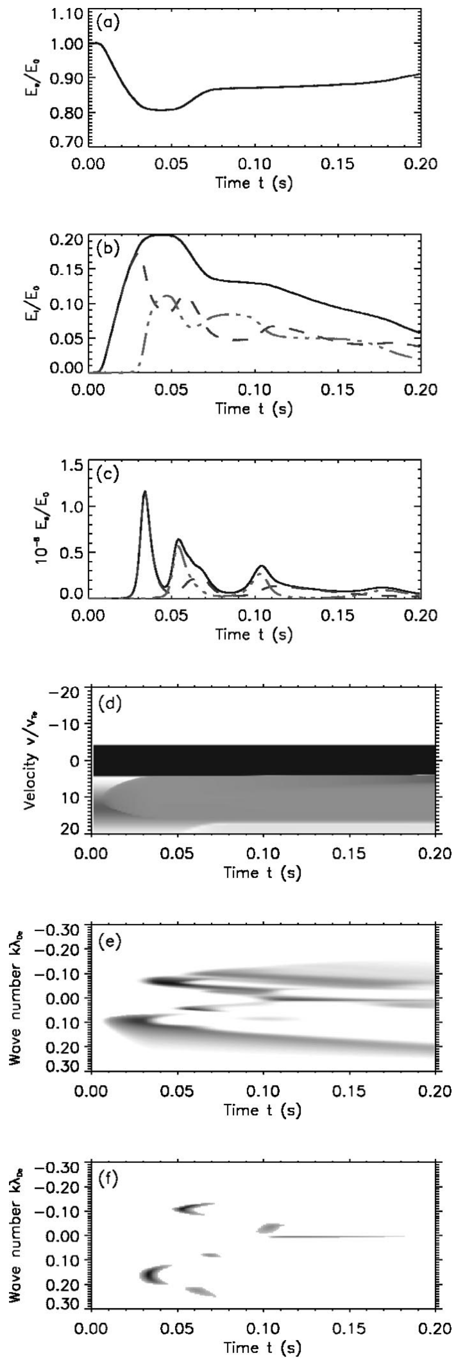

FIG. 7. The same as Fig. 1 but for a decreasing plasma density. Weak plasma inhomogeneity $L = -1\times10^9$ cm.

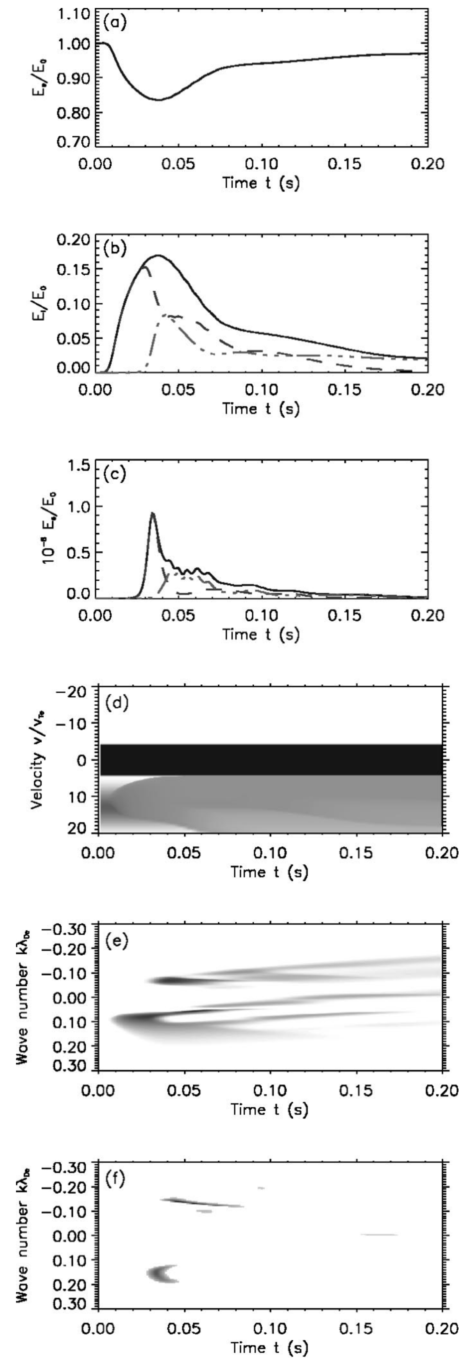

FIG. 8. The same as Fig. 1 but for an increasing plasma density. Weak plasma inhomogeneity $L = 1\times10^9$ cm.

densate. This high level of plasma waves accumulated near $k\approx 0$ is shifted by the plasma gradient out of the region where decay prohibited. As a result, Langmuir waves may again decay, producing ion-sound waves. This interplay of nonlinear processes leads to generation of sound waves with very small k. Since the damping rate is smaller for lower k, the intensity of these waves can be quite high (Figs. 7 and 8).

The plasma inhomogeneity also affects the spectral distribution of ion-sound waves. Comparing cases with homogeneous and inhomogeneous plasmas, we conclude that a plasma inhomogeneity increases the efficiency of ion-sound wave generation. The other interesting feature is the appearance of small wave number ion-sound waves in case of weak plasma gradients. The damping rate γ_k^s decreases with k, implying that long wavelengths are most likely to be observed late in the evolution of the turbulence. The distribution of these waves is a result of both nonlinear processes (decay and scattering) acting differently at low k.

IV. DISCUSSION AND MAIN RESULTS

As we see, due to a plasma gradient the Langmuir wave spectrum is shifted in k space with different processes being active. Thus, a plasma inhomogeneity is effective in switching between different processes in the system. Indeed, for sufficiently small L (strong gradient) the plasma inhomogeneity prohibits any nonlinear processes. Instead, the electron-Langmuir wave interaction becomes important. For $L>0$ the Langmuir waves are absorbed back by the beam, while for $L<0$ Langmuir waves are absorbed by thermal electrons via Landau damping. Increasing $|L|$ (reducing the gradient) we activated the next process, the three-wave decay of a Langmuir waves. Depending on plasma gradient, the Langmuir turbulence makes a fixed number of oscillations. The plasma inhomogeneity controls the rate of scattering off ions and decay at small k, where decay and scattering becomes equally important.

Our calculations demonstrate that the Langmuir wave spectrum in general reaches a quasistationary state more rapidly in an inhomogeneous plasma that in a homogeneous one.

The positive plasma gradient case shows that the amount of energy that is released in form of Langmuir waves is much smaller than that in case of a homogeneous plasma. The positive plasma gradient also leads to appearance of accelerated electrons in the plasma [25].

The ion-sound waves are heavily damped in plasmas with comparable ion and electron temperatures. First, this leads to a low level of ion-sound waves in comparison to that of Langmuir waves. Indeed, the energy accumulated in ion-sound waves is a small fraction of initial beam energy. Second, the ion-sound waves are seen as bursts with small time duration. Obviously, the plasma inhomogeneity can also influence the ion-sound turbulence. The interesting observation is that the period of time the bursts of ion-sound waves exist is dependent on plasma gradient. Generally, the bursts of ion-sound waves are longer in time in inhomogeneous plasma than in homogeneous. Therefore, Langmuir wave turbulence loses energy via ion-sound waves more intensively in comparison with the homogeneous plasma case. It may be that anomalous spectra of ion-sound waves observed by incoherent radar scattering can be explained in terms of decay from Langmuir waves [31], and we anticipate that naturally occurring plasma density gradients in the upper parts of the ionosphere can contribute also to these processes.

In view of application to astrophysical plasma, the plasma turbulence is normally observed via nonthermal radio emission [37,42,43]. The efficient emission mechanism giving radio emission at double plasma frequency is the coalescence of two Langmuir waves. In order to obtain high intensity of radio emission one has to supply high level of waves propagating at an angle to primary Langmuir waves. In this view low wave number Langmuir waves are the most effective. As shown, a plasma inhomogeneity successfully governs the evolution of the small wave number plasma waves, and provides us with a better understanding also of the Langmuir turbulence responsible for radio emission.

V. CONCLUSIONS

We demonstrated that a plasma inhomogeneity can act as a control parameter and play a crucial role in the development of weak Langmuir turbulence, by triggering various processes that affects the turbulence. The plasma inhomogeneity generally leads to a decline of turbulence, therefore, a steady state of weak turbulence is reached faster than in case of homogeneous plasma. Selective effects of Landau damping of Langmuir waves, or self-acceleration of electrons in a beam can easily be activated by proper choice of a density gradient.

The plasma inhomogeneity is the main process that is capable of switching between the decay and scattering off ions at very low wave numbers. This part of the Langmuir wave spectrum is important for radio emission, and a plasma inhomogeneity can, therefore, be significant in adjusting the flux of radio emission from the plasma. Generally, we expect that the radio emission should be stronger for inhomogeneous plasmas. It might be appropriate to mention that the relative importance of electron temperature gradients is negligible in comparison to density gradients with the same length scale L. When considering the last term in Eq. (5) we have an effective force acting on a wave packet being $\partial\omega_k/\partial x \sim \omega_{pe}/L$ for the density gradient, while it is $\partial\omega_k/\partial x \sim \omega_{pe}(k\lambda_{De})^2/L$ for the homogeneous density with inhomogeneous background electron plasma temperature. For this term in Eq. (5), a temperature gradient gives a negligible contribution as long as $(k\lambda_{De})^2 \ll 1$. The constraint implied in the WKB approximation (2) must be fulfilled also, but this is a rather weak requirement, considering the large values of L used in the present analysis. We must, however, keep in mind that both St_{decay} and St_{ion} are temperature dependent, and that temperature gradients can affect these terms.

The presence of inhomogeneity affects the decay process, and as a result ion-sound turbulence is generated more effectively. In particular, the plasma inhomogeneity can give rise to a quasisteady state of decay interaction, when the shift of the Langmuir wave spectrum due to inhomogeneity suppresses further developments of decay. From an application point of view, it is interesting that in our case a weak plasma inhomogeneity stimulates generation of ion-sound waves with very low wave numbers. We took particular care to consider physically realistic parameter values, in the present case some relevant for the solar corona. We find it of particular interest in this context, that the time scale for the initial evolution of the waves and the electron beam which is driving the process is short, of the order of 0.02–0.05 s, i.e., comparable to a typical electron-ion collision time, $\nu_{ei}^{-1} \geqslant 1/50$ s. A collisionless plasma model for this type of processes is, therefore, justified.

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